4.7 High energy and nuclear physics

Nuclear decay

Nuclear decay law	$N(t)=N(0)e^{-\lambda t}$	(4.163)	$N(t)$ number of nuclei remaining after time t t time
Half-life and mean life	$T_{1/2}=\dfrac{\ln 2}{\lambda}$	(4.164)	λ decay constant $T_{1/2}$ half-life
	$\langle T\rangle = 1/\lambda$	(4.165)	$\langle T\rangle$ mean lifetime
Successive decays $1\rightarrow 2\rightarrow 3$ (species 3 stable)			
	$N_1(t)=N_1(0)e^{-\lambda_1 t}$	(4.166)	N_1 population of species 1
	$N_2(t)=N_2(0)e^{-\lambda_2 t}+\dfrac{N_1(0)\lambda_1(e^{-\lambda_1 t}-e^{-\lambda_2 t})}{\lambda_2-\lambda_1}$	(4.167)	N_2 population of species 2
	$N_3(t)=N_3(0)+N_2(0)(1-e^{-\lambda_2 t})+N_1(0)\left(1+\dfrac{\lambda_1 e^{-\lambda_2 t}-\lambda_2 e^{-\lambda_1 t}}{\lambda_2-\lambda_1}\right)$	(4.168)	N_3 population of species 3 λ_1 decay constant $1\rightarrow 2$ λ_2 decay constant $2\rightarrow 3$
Geiger's law[a]	$v^3=a(R-x)$	(4.169)	v velocity of α particle x distance from source a constant
Geiger–Nuttall rule	$\log\lambda = b+c\log R$	(4.170)	R range b, c constants for each series α, β, and γ

[a]For α particles in air (empirical).

Nuclear binding energy

Liquid drop model[a]	$B=a_vA-a_sA^{2/3}-a_c\dfrac{Z^2}{A^{1/3}}-a_a\dfrac{(N-Z)^2}{A}+\delta(A)$	(4.171)	N number of neutrons A mass number ($=N+Z$) B semi-empirical binding energy Z number of protons a_v volume term (~ 15.8 MeV)
	$\delta(A)\simeq\begin{cases}+a_pA^{-3/4} & Z, N \text{ both even}\\ -a_pA^{-3/4} & Z, N \text{ both odd}\\ 0 & \text{otherwise}\end{cases}$	(4.172)	a_s surface term (~ 18.0 MeV) a_c Coulomb term (~ 0.72 MeV) a_a asymmetry term (~ 23.5 MeV) a_p pairing term (~ 33.5 MeV)
Semi-empirical mass formula	$M(Z,A)=ZM_H+Nm_n-B$	(4.173)	$M(Z,A)$ atomic mass M_H mass of hydrogen atom m_n neutron mass

[a]Coefficient values are empirical and approximate.

Nuclear collisions

Breit–Wigner formula[a]	$\sigma(E)=\frac{\pi}{k^2}g\frac{\Gamma_{ab}\Gamma_c}{(E-E_0)^2+\Gamma^2/4}$	(4.174)
	$g=\frac{2J+1}{(2s_a+1)(2s_b+1)}$	(4.175)
Total width	$\Gamma=\Gamma_{ab}+\Gamma_c$	(4.176)
Resonance lifetime	$\tau=\frac{\hbar}{\Gamma}$	(4.177)
Born scattering formula[b]	$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=\left\lvert\frac{2\mu}{\hbar^2}\int_0^\infty\frac{\sin Kr}{Kr}V(r)r^2\,\mathrm{d}r\right\rvert^2$	(4.178)
Mott scattering formula[c]	$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=\left(\frac{\alpha}{4E}\right)^2\left[\csc^4\frac{\chi}{2}+\sec^4\frac{\chi}{2}+\frac{A\cos\left(\frac{\alpha}{\hbar v}\ln\tan^2\frac{\chi}{2}\right)}{\sin^2\frac{\chi}{2}\cos\frac{\chi}{2}}\right]$	(4.179)
	$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\simeq\left(\frac{\alpha}{2E}\right)^2\frac{4-3\sin^2\chi}{\sin^4\chi}\quad(A=-1,\alpha\ll v\hbar)$	(4.180)

$\sigma(E)$	cross-section for $a+b\rightarrow c$
k	incoming wavenumber
g	spin factor
E	total energy (PE + KE)
E_0	resonant energy
Γ	width of resonant state R
Γ_{ab}	partial width into $a+b$
Γ_c	partial width into c
τ	resonance lifetime
J	total angular momentum quantum number of R
$s_{a,b}$	spins of a and b
$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$	differential collision cross-section
μ	reduced mass
K	$=\lvert\boldsymbol{k}_{\mathrm{in}}-\boldsymbol{k}_{\mathrm{out}}\rvert$ (see footnote)
r	radial distance
$V(r)$	potential energy of interaction
$\hbar$	(Planck constant)$/2\pi$
α/r	scattering potential energy
χ	scattering angle
v	closing velocity
A	$=2$ for spin-zero particles, $=-1$ for spin-half particles

[a]For the reaction $a+b\leftrightarrow R\rightarrow c$ in the centre of mass frame.
[b]For a central field. The Born approximation holds when the potential energy of scattering, V, is much less than the total kinetic energy. K is the magnitude of the change in the particle's wavevector due to scattering.
[c]For identical particles undergoing Coulomb scattering in the centre of mass frame. Nonidentical particles obey the Rutherford scattering formula (page 72).

Relativistic wave equations[a]

Klein–Gordon equation (massive, spin zero particles)	$(\nabla^2-m^2)\psi=\frac{\partial^2\psi}{\partial t^2}$	(4.181)
Weyl equations (massless, spin 1/2 particles)	$\frac{\partial\boldsymbol{\psi}}{\partial t}=\pm\left(\boldsymbol{\sigma}_x\frac{\partial\boldsymbol{\psi}}{\partial x}+\boldsymbol{\sigma}_y\frac{\partial\boldsymbol{\psi}}{\partial y}+\boldsymbol{\sigma}_z\frac{\partial\boldsymbol{\psi}}{\partial z}\right)$	(4.182)
Dirac equation (massive, spin 1/2 particles)	$(\mathbf{i}\gamma^\mu\partial\mu-m)\boldsymbol{\psi}=0$	(4.183)
	where $\partial\mu=\left(\frac{\partial}{\partial t},\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)$	(4.184)
	$(\gamma^0)^2=\mathbf{1}_4;\quad(\gamma^1)^2=(\gamma^2)^2=(\gamma^3)^2=-\mathbf{1}_4$	(4.185)

ψ	wavefunction
m	particle mass
t	time
$\boldsymbol{\psi}$	spinor wavefunction
$\boldsymbol{\sigma}_i$	Pauli spin matrices (see page 26)
$\mathbf{i}$	$\mathbf{i}^2=-1$
γ^μ	Dirac matrices: $\gamma^0=\begin{pmatrix}\mathbf{1}_2 & 0\\ 0 & -\mathbf{1}_2\end{pmatrix}$ $\gamma^i=\begin{pmatrix}0 & \boldsymbol{\sigma}_i\\ -\boldsymbol{\sigma}_i & 0\end{pmatrix}$
$\mathbf{1}_n$	$n\times n$ unit matrix

[a]Written in natural units, with $c=\hbar=1$.